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Application of Fracture Mechanics to Rubber Articles, Including Tyres

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Application of fracture mechanics to rubber articles, including tyres

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[Plate 1]

The use of a fracture mechanics approach, based on the rate of release of strain energy, to account for various features of the failure of vulcanized rubbers is outlined. The properties considered include those to which fracture mechanics is often applied – tear, tensile failure, crack growth and fatigue – and others to which its application is less usual – abrasion, ozone attack and cutting by sharp objects. The relation of macroscopically observed properties to the basic molecular strength of the material is also discussed. An example of a quantitative practical application of the rubber fracture work, to groove cracking in tyres, is then considered. Finally, the rather more complicated fracture that can occur in rubber–cord laminates is discussed and it is shown that the energetics approach can be applied to some features, at least, of this.

1. GENERAL INTRODUCTION

A fracture mechanics approach has proved extremely helpful in understanding strength and some other properties of vulcanized rubbers. The highly elastic, nonlinear behaviour of these materials points to the need for a rather general approach and one that has proved successful is based directly on the energetics concept originally proposed by Griffith (1920). Thus the rate at which strain energy is released by a growing crack is the parameter that has been widely used in the work on rubber. This was first suggested by Rivlin & Thomas (1953) in connection with tearing and for that reason the energy release rate has become known as the 'tearing energy', although the approach has subsequently been extended to many other properties. There have been two main departures from Griffith's ideas: (i) in recognizing that energy release rates could be calculated for various test pieces from general considerations irrespective of the large nonlinear deformations involved; (ii) in extending the fracture aspect of the concept from a single surface energy criterion for catastrophic failure to cover a wide range of phenomena including time-dependent and cyclic effects and also chemical attack. The energies involved in the failure phenomena to which fracture mechanics has been applied for rubber range from less than 10 μJ/cm² to about 10 J/cm². It was also recognized at an early stage (Thomas 1955), and again from general arguments not involving assumptions of linear elasticity, that the overall strain energy release rate is closely related to the strain energy density at the crack tip. The latter, of course, appears likely to provide a more fundamental measure of the onset of fracture. This introduction continues with a description of the basis of the approach used for rubbers by reference to the original work on tearing.

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1.1. *Tearing of rubbers*

The application of fracture mechanics to rubbers had its origins in the observation that conventional tear tests, which use different test pieces, often rate a given series of vulcanizates in different orders. To try to overcome this difficulty, Rivlin & Thomas (1953) proposed a criterion based on the strain energy release rate and defined mathematically by

$$T = -\partial U/\partial A,$$

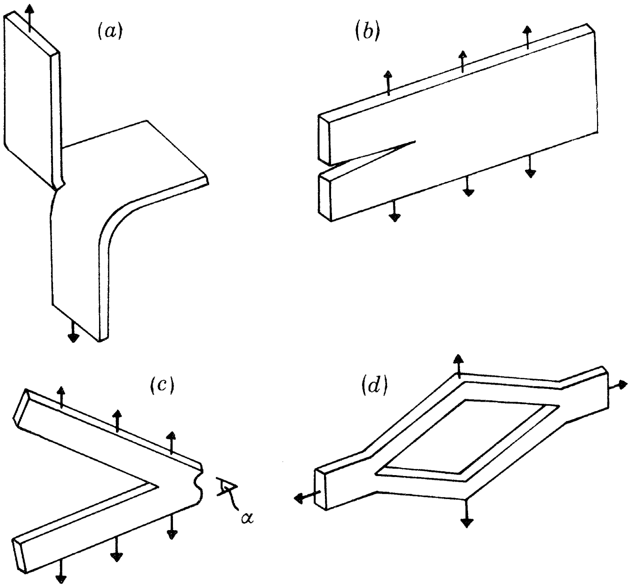


FIGURE 1. Some of the test-pieces that have been used for tear and other fracture measurements on rubber: (*a*) 'trousers'; (*b*) pure shear; (*c*) angled (in which the angle α is held constant); (*d*) split tear.

where $\partial U/\partial A$ represents the change in the elastic strain energy U stored in a specimen due to unit increase in area A of one fracture surface of a crack, the partial derivative indicating that the specimen is considered to be held at constant deformation, so that the external forces do no work. The 'tearing energy', T, is thus directly analogous to the energy release rate derived by Griffith; however, in the tearing of rubber, energy is dissipated irreversibly so that T is greater, generally much greater, than the surface free energy. Nevertheless, when tear results are expressed in terms of T they are independent of the test piece used, for test pieces of widely differing shapes and types of deformation (figure 1).

As indicated above, the energy release rate is closely related to the stress concentration at a crack tip. For a model crack having a semicircular tip of diameter d, Thomas (1955) showed that

$$T = d\int_0^{\frac{1}{2}\pi} W_\phi \cos\phi \,\mathrm{d}\phi, \tag{1}$$

where W_ϕ is the elastic energy stored per unit volume in the surface layer of the tip at an angle ϕ from the pole (figure 2). This equation was derived without making any assumptions about the stress–strain behaviour of the material and was verified experimentally. Furthermore, if the average strain energy density of the tip is W_t, the following approximation is found to hold:

$$T \approx W_\mathrm{t} d. \tag{2}$$

The tearing energy approach was found to work well for both crystallizing and non-crystallizing rubbers (Rivlin & Thomas 1953; Thomas 1960), although the tear characteristics differ markedly, being strongly rate- and temperature-sensitive for non-crystallizing rubbers but insensitive over wide ranges of these variables for strain-crystallizing rubbers (Greensmith & Thomas 1955; Greensmith 1956). The behaviour of non-crystallizing rubbers shows close correlation with the variation in their viscoelastic properties with rate and temperature, suggesting that viscoelastic hysteresis makes a major contribution to tear strength (Mullins 1959; Gent and Henry 1967). For crystallizing rubbers, viscoelastic effects appear to be swamped by the crystallization, which can induce very marked hysteresis at high strains.

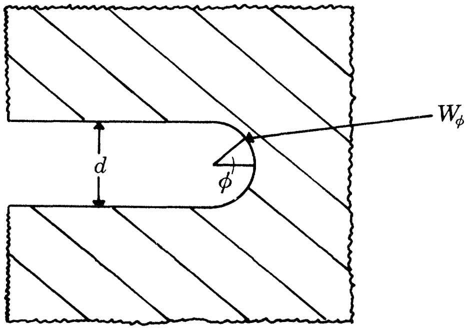


FIGURE 2. Model crack tip. W_ϕ is the strain energy density at the surface at angle ϕ.

1.2. *Crack growth*

Small-scale crack growth can occur in rubbers, as in other materials, at energies below those required for tearing or catastrophic failure. This applies particularly under repeated stressing, although in non-crystallizing rubbers similar crack growth (but of a time-dependent nature) can also occur at constant stress. In either case the crack growth may ultimately lead to fatigue failure and these processes have been studied in some detail. For repeated stressing, it is found that if the amount of crack growth per cycle is plotted against the maximum tearing energy attained during the cycle, the results are independent of the form of test piece used (figure 3*a*). Thus the tearing energy concept is also valid for small-scale cyclic crack growth. A similar conclusion applies to time-dependent growth, when this occurs.

The tear strength represents an upper bound for small-scale growth. In the region below this the cyclic crack growth behaviour can often be approximated by relations of the form

$$\mathrm{d}c/\mathrm{d}n = BT^{\beta}, \tag{3}$$

where c is the crack length, n the number of cycles and B and β are constants. For most vulcanizates β lies between about 1 and 6, depending particularly on the elastomer used and to a lesser extent on other factors such as the degree of cross-linking; for example, for natural rubber vulcanizates β is often about 2 (Thomas 1958; Gent *et al.* 1964), whereas for vulcanizates of the non-crystallizing styrene–butadiene rubber (SBR) β is commonly in the region of 4 (Lake & Lindley 1964).

At lower tearing energies, the power law relations break down and for many elastomers there is a linear region with an intercept on the T axis that represents the energy (T_0) required for the initiation of mechanical crack growth. Below T_0 only very slow growth occurs at a rate that is essentially independent of tearing energy and this growth is entirely attributable to the effects of chemical attack by ozone. Thus in this region the crack growth characteristics are generally of the form

$$\mathrm{d}c/\mathrm{d}n = H(T-T_0)+r_{\mathrm{z}}, \tag{4}$$

where the first term on the right-hand side represents the mechanical growth, H being a constant, and r_{z} represents the rate of ozone crack growth.

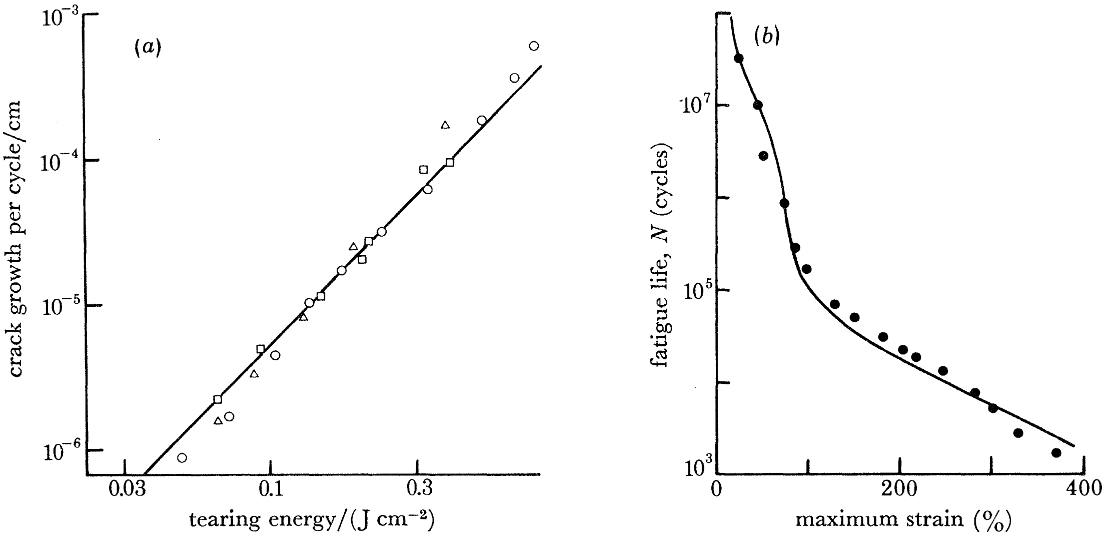


FIGURE 3. Crack growth and fatigue (unfilled natural rubber vulcanizate). (*a*) Cyclic crack growth rate plotted against maximum tearing energy of the cycle for different test pieces: □, 'trousers'; △, pure shear; ○, tensile strip (with an edge crack). (*b*) Tensile fatigue life plotted against maximum strain of the cycle (minimum strain zero throughout): ●, averages of 4–8 experimental results; —, theoretical curve derived from the crack growth characteristics assuming a natural flaw size of 25 μm.

The magnitude of T_0 is of similar order, *ca.* 5 mJ/cm², for vulcanizates of a range of elastomers including natural rubber, SBR, polychloroprene and butyl rubber, which differ widely in other strength properties such as tear and tensile strengths (Lake & Lindley 1965). The similarity of T_0 for different elastomers under various conditions suggests that this property may be governed fairly directly by the primary carbon–carbon bond strength. An estimate of the minimum tearing energy required to produce mechanical bond rupture can be made by using (2) if it is assumed that the tip diameter has its smallest possible value, which for an elastomer is of the order of the distance between cross-links in the unstrained state; this gives $d \approx \xi\mu^{\frac{1}{2}}$, where ξ is the length of a monomer unit and μ the number of monomer units between cross-links. Assuming forces to be transmitted primarily via the cross-links, the energy stored by the carbon–carbon double bonds will be small at the breaking force of the single bonds, so that the maximum possible energy density will be of the order of bJ, where b is the number of single bonds per unit volume and J is the energy stored by each single bond at its rupture point. Substitution in (2) yields

$$T_0 \approx bJ\xi\mu^{\frac{1}{2}}. \tag{5}$$

For a typical natural rubber vulcanizate the following approximate values apply: $\xi = 5\times10^{-8}$ cm, $b = 2.4\times10^{22}\ \mathrm{cm}^{-3}$, $\mu = 100$; putting $J = 3.3\times10^{-19}$ J (the dissociation energy of the weakest C—C bond in the isoprene unit), equation (5) gives $T_0 \approx 4\ \mathrm{mJ/cm}^2$. More precise calculation (Lake & Thomas 1967) yields about half this value, which is nevertheless in good agreement with the experimental results in view of various uncertainties. Variations between different elastomers would be expected to be fairly small on this basis, as is observed. Thus T_0 can be related approximately to the primary bond strength and molecular structure of a vulcanizate.

When T_0 is exceeded, the occurrence of crack growth means that there are regions in the vicinity of a crack tip where retraction of material occurs. Thus the mechanical hysteresis of the material will influence the stress concentration at the tip and hence contribute to the complex picture that applies to crack growth at higher severities. In particular, tear strength and tensile strength (to be discussed in §1.4), which represent the upper bounds of the macroscopic failure spectrum, do not appear likely to be amenable to straightforward, if any, fundamental interpretation.

1.3. *Fatigue*

Fatigue life can be predicted from the crack growth behaviour if it is assumed that crack growth initiates at small flaws. In the case of an edge crack in a tensile specimen, the tearing energy is given by (Rivlin & Thomas 1953)

$$T = 2KWc, \tag{6}$$

where W is the strain energy density in the bulk of the specimen and K a slowly varying function of strain, which has been determined independently (Greensmith 1963). At small strains, K approaches a value in the region of π so that (6) gives essentially the same energy release rate as that derived originally by Griffith (1920).

For power law relations of the form given by (3), simple integration, substituting from (6) for T, gives the fatigue life as

$$N = \{(\beta-1)B(2KW)^{\beta}c_0^{\beta-1}\}^{-1}, \tag{7}$$

where c_0 is the effective initial flaw size. This equation is found to work well for the strain dependence of fatigue life at moderate and high strains and the flaw size required for quantitative agreement, typically a few thousandths of a centimetre, is consistent with observable imperfections in rubber due to particulate impurities or moulding or die-stamping imperfections (Gent 1962; Lindley & Thomas 1962; Gent *et al.* 1964; Lake & Lindley 1964). This approach can be extended to lower severities, where the power law relations break down, by using (4) instead. Figure 3*b* shows fatigue results over a wide range and compares them with the theoretical relation derived from the crack growth characteristics. The sharp upturn in the fatigue curve corresponds to T_0 and constitutes a mechanical fatigue limit below which failure is initiated solely by ozone (Lake & Lindley 1965). Fatigue failure under constant load or at constant deformation can similarly be treated for rubbers that show time-dependent crack growth (Lake & Lindley 1964).

1.4. *Other failure phenomena*

The fracture mechanics methods used for rubber have found application to several other phenomena in addition to those mentioned so far. The most obvious extension is to tensile strength. In essence, tensile failure can be regarded as a one-cycle fatigue test in which, for non-crystallizing rubbers, failure occurs when the tear strength is reached, although there are complications attributable to scale effects in the crack growth (Greensmith 1964; Lake & Yeoh 1980). For crystallizing rubbers there are further complications when the bulk of the test-piece becomes crystalline (as normally happens before tensile failure), since failure does not then occur at the normal tear strength and stable crack growth is observed at higher tearing energies.

A modified theory appears to take account of this fairly successfully (Thomas 1966; Thomas & Whittle 1970), but because of these complications tensile strength is a difficult property to interpret.

Chemical attack by ozone has already been mentioned as a contributory factor in fatigue at low strains. In addition, ozone can cause cracking in rubber held at constant deformation. Indeed, for rubbers that do not contain added protective agents, such cracking commonly has a threshold strain of only a few per cent extension. Experiments on test-pieces containing inserted cuts indicate that the cracking process has an energy requirement of about 10 μJ/cm^2 (Braden & Gent 1960). By using this energy and the observed threshold strain, a flaw size of the order 10^{-3} cm is required to satisfy (6); as discussed above, this is plausible for rubber and furthermore agrees well with the independent estimate from fatigue data. Thus the fracture mechanics approach enables the threshold condition for chemically induced cracking by ozone to be satisfactorily accounted for.

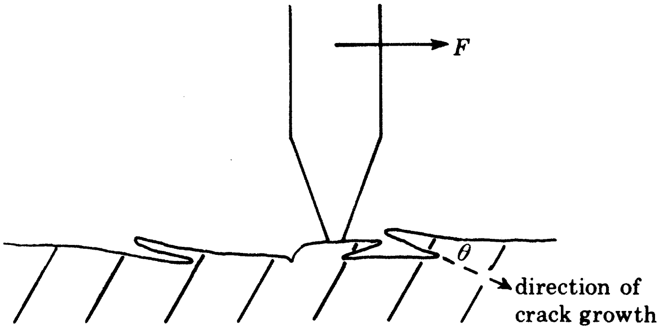


Figure 4. Sketch of part of the profile of an abrasion pattern formed by a razor blade (with its length perpendicular to the plane of the figure) under the action of an abrading force F.

Abrasion is another phenomenon that has been successfully treated by the energetics approach. Patterns consisting of small tongues of rubber often develop during abrasion, and the production of these tongues is attributable to crack growth. For an idealized, uniform pattern formed by an abrading force F per unit width acting on a line abrader (figure 4) it can be shown (Southern & Thomas 1978) that the tearing energy is given by

$$T = F(1+\cos\theta), \tag{8}$$

where θ is the angle that a crack makes with the surface. It is assumed that abrasion occurs because the tongues break off; since the crack length does not appear in (8), the detailed way in which this happens – i.e. whether in a single cycle or many cycles – does not matter. Thus if the cracks advance a distance x for each pass of the abrader (i.e. each cycle) the average abraded volume per unit area of surface per cycle will be

$$v = x\sin\theta, \tag{9}$$

where x, and hence v, can be calculated from the crack growth characteristics of the rubber (cf. (3), for example) and equation (8). Comparison of calculated and measured abrasion rates shows good agreement for various non-crystallizing rubbers; agreement for strain-crystallizing natural rubber is less good, the abrasion being faster than predicted, suggesting that crystallization either does not occur or is ineffective during abrasion.

A fracture mechanics approach has also proved useful in connection with studies of the cutting of rubber by sharp objects (Lake & Yeoh 1978). Cutting is of importance in service, for example for tyres or conveyer belting, primarily under wet conditions when friction between the rubber and the cutting object is much reduced (otherwise the frictional forces tend to be very large and cutting correspondingly difficult). In order to study the process under conditions where frictional effects were largely absent, a method was developed in which the cutting implement, a razor blade, was applied to the tip of a crack in a stretched tear test-piece. The tearing energy approach again enables results from differently shaped test-pieces and different deformations to be superimposed. Furthermore, in a region of low tearing energy, rapid 'catastrophic' cutting is found to occur when the total energy available, from both the deformation of the rubber and the force applied to the blade, reaches a constant value. This applies for all vulcanizates examined, although the magnitude of the constant varies for different vulcanizates.

Finally, similar energetics methods have proved useful in tackling sliding friction under conditions where waves of detachment (Schallamach waves) are formed and also various adhesion problems (see, for example, the reviews by Gent & Hamed (1978) and Roberts (1979)).

2. TREAD GROOVE CRACKING IN TYRES

Cracks sometimes develop in the base of the tread grooves in tyres (see figure 5*a*) and can lead to failure. A study of this groove cracking process provides an example of a quantitative practical application of the fracture mechanics work carried out on rubber. A major difficulty in applying the work quantitatively is to determine the strain energy release rate for cracks in articles used in practice, which are often of complex shape and structure and can undergo complex deformations. For groove cracking, a special technique was developed that enabled the tearing energy and the strain in a tyre groove to be estimated (Clapson & Lake 1970). This technique involves measurement of the amount by which a crack opens under stress (note that this is the *elastic* opening of the crack and is not to be confused with the crack opening displace-

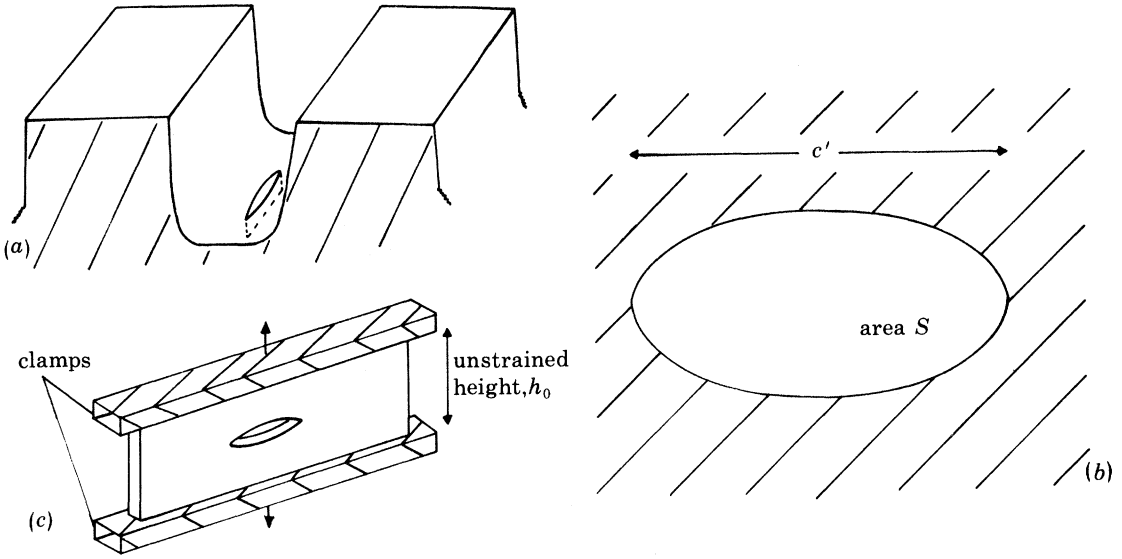


FIGURE 5. Groove cracking in tyres. (*a*) Sketch of a section of a tyre tread illustrating the location of a typical groove crack. (*b*) Parameters involved in the crack opening method for determining the strain energy release rate. (*c*) Pure shear test-piece containing a central crack.

ment). From consideration of the work required to close up an open crack it can be shown that for a crack in a sheet the tearing energy is given approximately by

$$T \approx \tfrac{1}{2}\sigma\, \partial S/\partial c', \tag{10}$$

where σ is the stress referred to the unstrained state, S the area of the crack opening, c' the crack length in the strained state (figure 5*b*) and the partial derivative indicates, as before, that the specimen is considered to be held at constant deformation. This method was checked by experiments on model test-pieces, deformed in simple extension or pure shear, and found to work well. Indeed it was thought that the pure shear test-piece containing a central crack (figure 5*c*) might be a good model for a crack in a tyre groove and in fact this turned out to be so. Figure 6*a* compares the tearing energy relations (in reduced variables form) obtained from experimental measurements on cracks of different lengths by using equation (10) and it can be seen that they are in close agreement. The tyre results are for relatively deep cracks (*ca.* 4 mm), but the approach can also be used for shallow cracks, when the depth rather than the length can become the controlling factor and can be similarly treated.

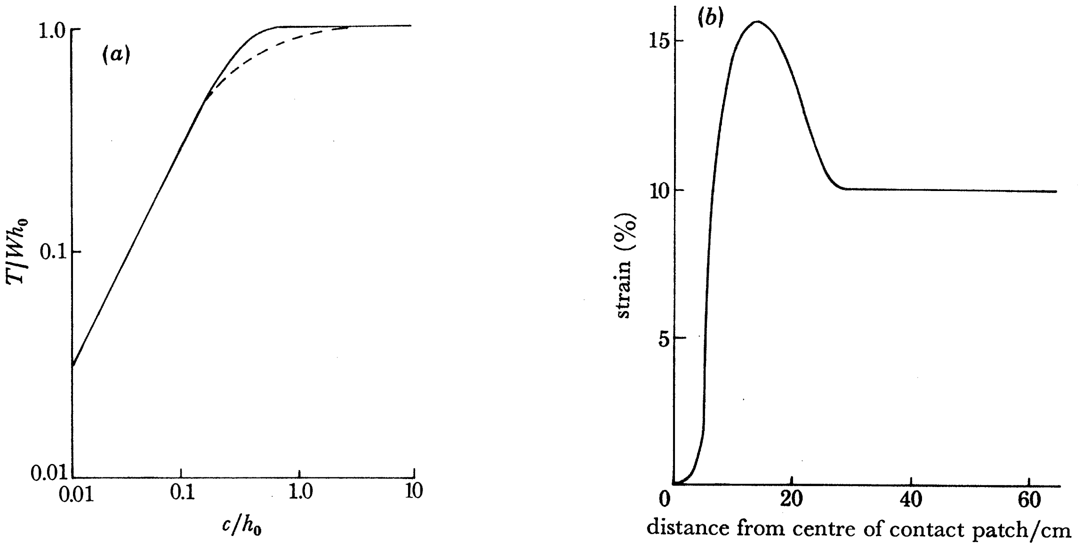


FIGURE 6. Use of crack opening method. (*a*) Results obtained for the tearing energy T as a function of crack length c for a crack in a pure shear test-piece (—) and a deep crack in a tyre groove (– – –). W is the strain energy density in the bulk of the rubber (i.e. away from the crack) and h_0 the unstrained height for the pure shear test-piece or width for the tyre groove. (*b*) Variation of lateral tensile strain in a groove with distance from the centre of the contact patch, for a lorry tyre under load.

The crack opening technique made it apparent that a lateral tensile strain exists in the grooves of an inflated tyre even when the tyre is not under load. Loading has two effects on the strain (figure 6*b*): (i) near the area of contact with the road the strain increases somewhat; (ii) within the contact patch itself the strain falls sharply, becoming zero for the particular tyres studied. By using the measured strain cycle, the tearing energy relation for a crack in a groove shown in figure 6*a* and the independently measured crack growth characteristics of the tread rubber obtained in the laboratory, the rate of growth of groove cracks can be predicted. Figure 7 compares the predicted behaviour with that observed in tyres running in normal service and under test on a rig; a groove strain cycle of 0–9% was chosen to give best fit (there being no other adjustable parameters). This compares with an observed average strain cycle of about 0–11 %, which is a good agreement in view of various uncertainties.

The theoretical curve in figure 7 shows an intercept on the abscissa at a crack length of about 1 mm. This corresponds to the mechanical crack growth limit T_0 referred to above and suggests that some form of initiation is required for groove cracking to occur. Consistent with this, no cracking was observed in rig tests unless cuts greater than 1 mm in length were inserted. In service, cutting by flints or other sharp objects can be a source of initiation, and ozone cracking is another possibility.

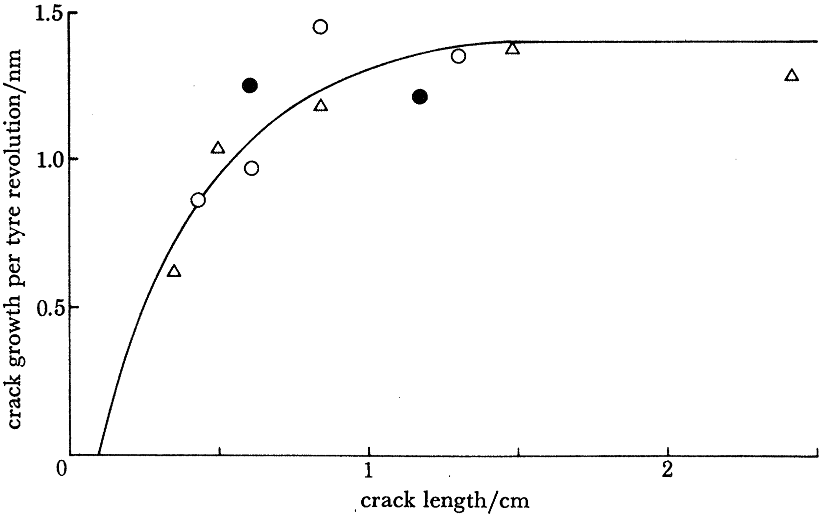


Figure 7. Comparison of observed (points) and predicted (curve) groove cracking behaviour. Experimental results from service tests (on buses running in the city of Bath, Somerset): ○, natural cracks; ●, inserted cuts. Results from rig tests: △, inserted cuts.

3. Fatigue in rubber–cord laminates

Rubber–cord laminated structures are used in tyres, conveyer belting and other rubber articles. In common with other laminates, failure in these structures can be complex and can occur, for example, because of loss of adhesion between the cords and the rubber or through various forms of cracking. In the work described below, two forms of cracking are considered: (i) growth of cracks between the ply layers and (ii) growth of roughly cylindrical cracks around individual cords: 'socketing'.

3.1. *Laminate deformations and failure mechanisms*

The type of structure considered consists of a sheet of rubber containing two layers of symmetrically disposed cords that are strongly bonded to the rubber. Under an overall extension, such a test-piece can broadly be divided into three deformation regions (figure 8*a*): a central region where the deformations are relatively uniform and obey approximately a pantographing model if the ply layers are close together; a region along either edge where they vary in a complex manner and where large shear strains can develop, particularly near the cord ends. Depending on the detailed design, shear strains up to 1000 % can occur (for overall extensions up to 5 %) so that, not surprisingly, cracks can initiate very rapidly in these regions. They may then grow between the ply layers both along the length and across the width of a specimen.

Since there are many cord ends, there are many potential sites for initiation along the length, so that growth in this direction proceeds relatively easily and rather erratically as many cracks amalgamate to form one long inter-ply crack. The growth in the transverse direction is better defined and less erratic and it is this that is believed to be the main factor influencing the life of a specimen when inter-ply cracking occurs (Breidenbach & Lake 1979).

Alternatively, cracking sometimes occurs *around* individual cords rather than between the ply layers. This 'socketing' process is readily distinguished from inter-ply cracking since the cord ends can be seen to move in and out of the rubber as a specimen is extended and retracted. Such failure could occur because of loss of adhesion but, as will be discussed, in this instance it is believed to be the result of crack growth.

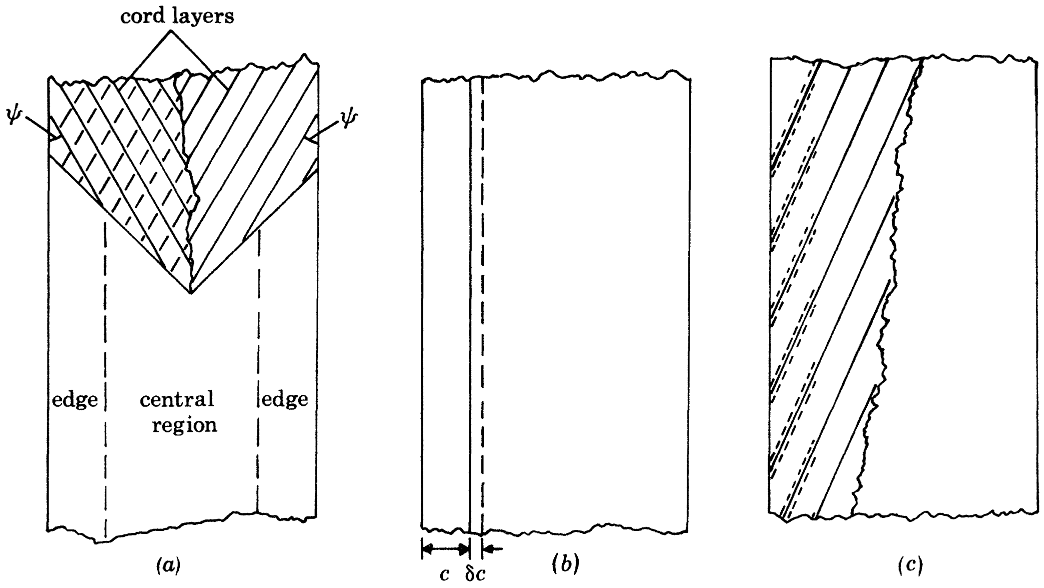


FIGURE 8. Schematic diagrams illustrating (*a*) a symmetrical two-ply laminate (with cord angles $\pm\psi$) and the three deformation regions, (*b*) an 'ideal' inter-ply crack of length c with a propagation step δc, and (*c*) the development of cylindrical cracks around individual cords ('socketing').

3.2. *Theory*

In inter-ply cracking, an idealized crack is considered that is of uniform length c along the test-piece (figure 8*b*). It is assumed (i) that as the crack advances, the complex deformation region at the edge of the test-piece moves with it, and (ii) that the crack is sufficiently long and the stiffness of a single ply layer sufficiently small for the strain energy in the rubber at the rear (edge) end to be relaxed essentially to zero. Under these circumstances the propagation of the crack by an amount δc (figure 8*b*) results in the transfer of a volume of material $t\delta c$ per unit length from the central region to the edge, where it is essentially undeformed. Thus the energy lost in the process, δV per unit length, is given by

$$\delta V = W_c t \delta c,$$

where W_c is the strain energy density in the central region. By definition, $T = \delta V/\delta c$ for this case, so that the tearing energy is given by

$$T = W_c t. \tag{11}$$

If, alternatively, failure results from cracking around the individual cords, then again an ideal case in which this 'socketing' is uniform along the test piece is considered (figure 8*c*). Similar assumptions to those above are taken to apply if socketing develops in *one* of the ply layers. Thus the energy released by a transverse propagation δc (cf. figure 8*b*) is the same as for an inter-ply crack, but in socketing the total area of crack growth per unit length of the test piece is

$$\delta A = \pi D m \delta c,$$

where m is the number of cords per unit length (perpendicular to the cord direction) and D the diameter of a crack around a cord. Putting $T = \delta V/\delta A$ yields

$$T = W_c t/\pi m D. \tag{12}$$

Comparison of (11) and (12) indicates that the energy available for crack growth will be the same for the two mechanisms when

$$m = 1/\pi D = m^*, \text{ say.} \tag{13}$$

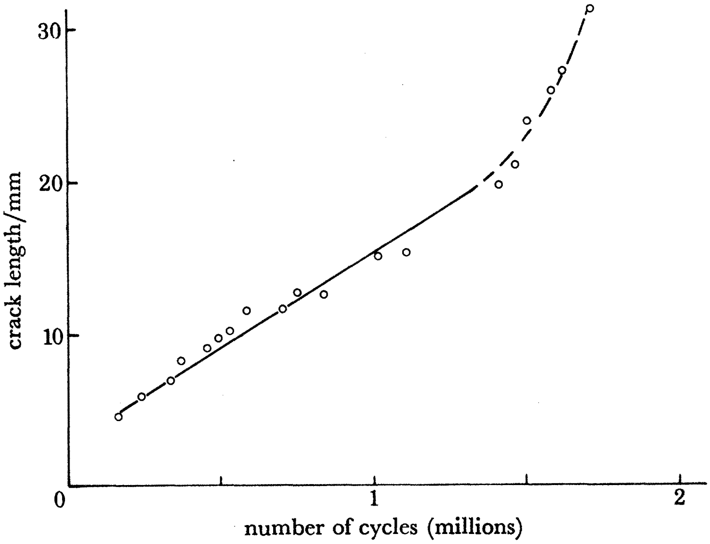


FIGURE 9. Transverse crack length plotted against number of cycles for an inter-ply crack in a laminate subjected to repeated tensile deformations of fixed amplitude.

3.3 *Results*

Figure 9 shows an example of typical interply crack growth in a laminate subjected to repeated extensions of fixed amplitude. The growth can be divided into three regions:

(i) an initiation region (for which no points are shown) during which cracks begin to grow in the vicinity of cord ends and join up in the longitudinal direction; depending on the detailed design, the rate in this region may be faster or slower than subsequently;

(ii) a stable growth region when the longitudinal crack length has become much larger than the transverse crack length and where the latter is at least of the order of the test-piece thickness; in this region the transverse growth rate remains essentially constant;

(iii) a final phase when cracks have grown across a substantial proportion of the test-piece and when the bulk deformation in the unfailed regions is appreciably non-uniform; this can cause either an increase or a decrease in the transverse growth rate, depending on the detailed configuration of the cracks.

In the work described here, measurements were made mainly in the stable growth region, and the theory given above relates to this region. It may be noted that the constancy of the growth rate in this region is consistent with the form of (11) according to which the tearing energy does not depend on crack length. For a given test-piece, graphs such as that shown in figure 9 can normally be obtained from several cracks. Figure 10 shows averaged steady-state rates of growth in the laminates (and the ranges) for various fixed maximum strains ranging from about 2 % to about 5 % (the minimum strain being zero throughout); the tearing energies were calculated by using (11), except that the strain energy density was the average for the test-piece as a whole rather than that in the central region, which cannot be measured directly.

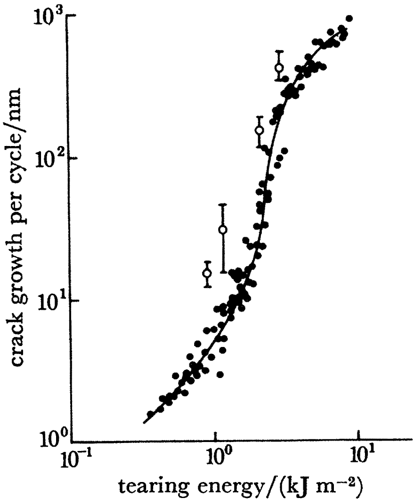
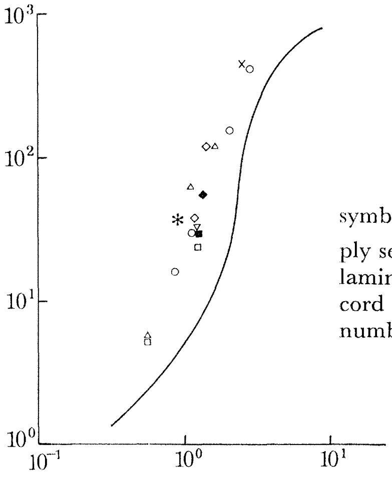


FIGURE 10. Comparison of averaged crack growth rates in laminates of standard construction (○) with results from independent measurements on tensile strip test-pieces of the ply rubber (●). The bars indicate the range of the results for the laminates.

Also shown in figure 10 are the independently measured crack growth characteristics of the ply rubber obtained from experiments on tensile strip test-pieces of the rubber alone containing edge cracks (by using (6) to calculate T). The two sets of results are in fairly good agreement and show a similar dependence on T.

The results in figure 10 were obtained by using a standard test-piece thickness, ply separation and cord angle. The latter two variables both affect the test-piece stiffness – very strongly in the case of the cord angle – while according to (11), the former should affect the tearing energy directly. Figure 11 shows results in which all three variables were altered; as can be seen, the use of the tearing energy superposes all the results onto a single relation. Also shown in figure 11 are results for test-pieces in which the cord density was reduced from the standard 10 ends per inch (perpendicular to the cord direction) to 5 and $3\frac{1}{3}$ ends per inch respectively (1 in = 2.54 cm). In both cases, failure occurred in these test-pieces by socketing rather than by inter-ply cracking (which was the primary cause of failure in all other cases). This change in mechanism could be seen strikingly, since once the cracking became appreciable, the ends of socketed cords disappeared into the rubber on extending the test-piece. Because of the enclosed nature of the crack it is not easy to measure the rate of growth of a socket, but figure 11 shows



symbol	○	△	▽	□	◇	■	◆	×	✻
ply separation/mm	1.1	2.0	2.3	2.0	3.9	1.3	1.3	1.3	1.3
laminate thickness/mm	6.9	3.4	7.0	9.7	6.9	3.5	3.5	6.2	6.5
cord angle, ψ/deg	22	22	22	22	22	30	15	22	22
number of cords per inch	10	10	10	10	10	10	10	5	$3\frac{1}{3}$

FIGURE 11. Averaged crack growth rates for laminates of various constructions. The line is the same as that drawn through the independently measured crack growth characteristics shown in figure 10.

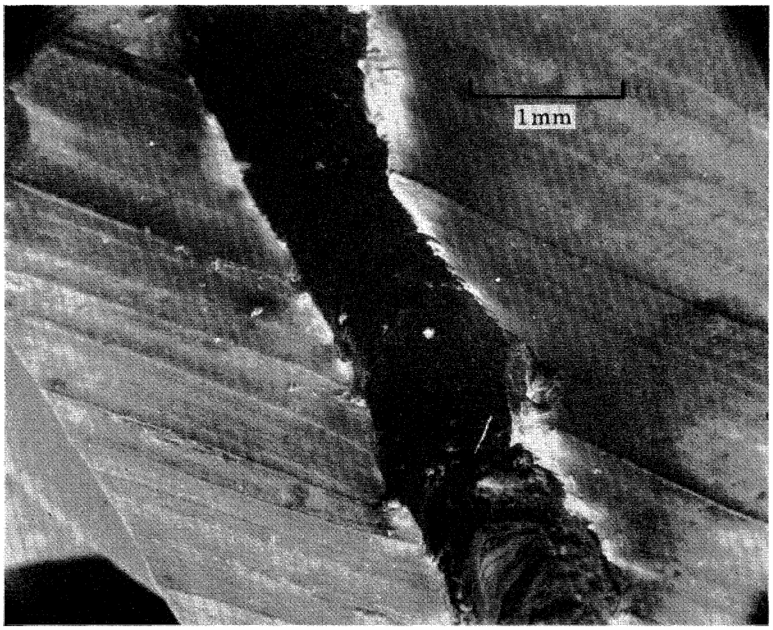


FIGURE 12. Low-magnification electron micrograph showing part of a socket formed by the growth of a roughly cylindrical crack around an individual cord.

average rates estimated by dividing the final socket length (obtained by cutting open a test-piece at the end of a test) by the number of cycles during which appreciable socketing had been observed. These are plotted against the tearing energy calculated from (12), but with the use of the average strain energy density, as for inter-ply cracking. The socket diameter D was taken as the diameter defined by the outer wire of the steel cord bundle (just over 1 mm). Examination of sockets after failure indicated this to be a reasonable estimate, as figure 12, plate 1, illustrates. With failure occurring so close to the cord it might be supposed to be interfacial (i.e. adhesive) in nature. Indeed, in the present case most of the length of a wire in a socket was bright steel without a rubber (or brass) covering. However, close to the tip, the wire was observed to be rubber-covered. This, together with the agreement with the other results shown in figure 11, suggests that failure was due to crack growth in the rubber in this case and that the exposure of the steel was caused by subsequent abrasion.

According to (13), the transition from inter-ply failure to socketing should occur when the cord density falls to a value m^* of $1/\pi D$. This corresponds to about 8 ends per inch for the cord used, in agreement with the experimental observation of between 5 and 10 ends per inch. Thus the theory appears fairly successful in predicting both the transition to socketing and the rate of socket growth; it also indicates that there is a region immediately beyond the transition where the overall failure rate will be slower if socketing occurs, owing to the angle at which the cracks grow.

4. Conclusions

The work described has shown that a simple fracture mechanics approach based on the strain energy released by a growing crack can be used successfully in relation to a number of properties of rubber. This applies not only to 'conventional' strength properties such as tearing, crack growth, fatigue and tensile failure, but also to other properties including ozone attack, abrasion and cutting by sharp objects. This diversity of application is perhaps one respect in which the work on rubbers has differed from that on other materials. Another difference has been in the variety of test-pieces employed. The high extensibility and flexibility of rubbers is possibly an advantage in this respect. The energy release rate allows results for all the different test-pieces to be correlated, but the ability to use different test-pieces has sometimes been important in enabling certain aspects of behaviour to be unravelled.

The fracture mechanics approach has enabled service behaviour to be predicted from laboratory measurements, as in the groove cracking case described. For rubber–cord laminates, the similarity of the crack growth results to the independently measured crack growth characteristics of the rubber suggests that the energetics method is also applicable to the complicated behaviour of these materials. The broad agreement obtained appears to constitute strong evidence that for the materials and test conditions used, the failure is cohesive rather than adhesive in nature.

Some insight has been obtained into the molecular interpretation of the strength properties of rubber. Although difficult to measure, the onset of small-scale crack growth appears to be the simplest strength property to interpret from a fundamental viewpoint. At higher severities properties become increasingly influenced by additional factors, such as internal viscosity, and are correspondingly difficult to interpret.

In view of the simplicity of the energetics approach, it is perhaps remarkable that it has been successful in so many different ways.

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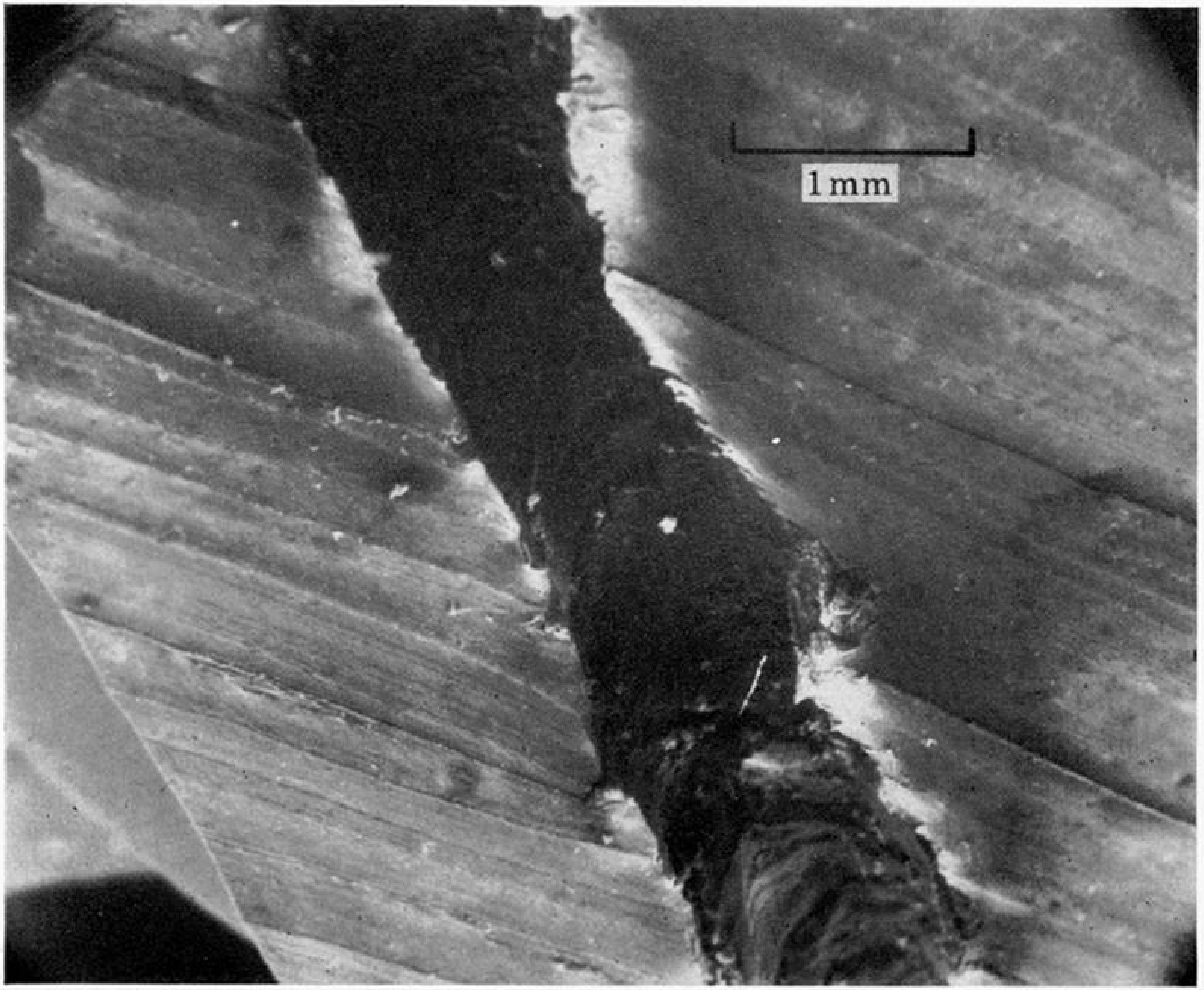

GURE 12. Low-magnification electron micrograph showing part of a socket formed by the growth of a roughly cylindrical crack around an individual cord.